A nonlinear quasiperiodic Kronig-Penney model

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Abstract

We introduce a quasiperiodic nonlinear Kronig-Penney model with a sequence of barrier heights constructed according to the Fibonacci inflation rule. We find that nonlinearity enhances transparency and reduces the localization properties of the corresponding linear quasiperiodic Kronig-Penney model.

Recent advances in nanodevice manufacturing have increased the interest in quasiperiodic one-dimensional models [1]. Quasiperiodic models provide a bridge between the regular ordered lattices of perfect materials and random lattices of amorphous systems. Interest in electronic propagation in quasicrystals has led to a thorough investigation of the band structure of some quasiperiodic systems, such as Penrose tiling and Fibonacci lattices [2-4]. When dielectric materials are used, novel phenomena such as photonic band gaps in the propagation of electromagnetic (EM) waves through the medium are possible [5-8]. In an approximation where only the scalar nature of the EM wave is taken into account, wave propagation in a periodic or disordered medium resembles the dynamics of an electron in a crystal lattice. As a result photonic bands and gaps arise in the periodic lattice case whereas EM wave localization is theoretically possible in the quasiperiodic and disordered cases [9,10]. In the present Letter we will address the issue of EM wave propagation in a quasiperiodic superlattice model where the medium has in addition nonlinear properties arising from the optical Kerr effect [10-12]. We will study the interplay between nonlinearity and quasiperiodicity and demonstrate that it can be used advantageously in wave propagation in nanodevices. The results obtained are also applicable to problems of electronic propagation in superlattices where the effective nonlinearity stems from many-body interactions [13].

We consider the following nonlinear Kronig-Penney equation,

$$E\psi(z) = - \frac{d^2\psi(z)}{dz^2} + \sum_{n=1}^{N} \lambda_n \delta(z-n)|\psi(z)|^2\psi(z).$$

(1)

In Eq. (1), $\psi(z)$ represents an electronic wavefunction or is the complex amplitude of a incoming plane wave with energy (or frequency) $E$ along direction $z$, and $\lambda_n$ is a coefficient that, in the case of the nonlinear optical medium, incorporates the nonlinear susceptibility $\chi^{(3)}$ and the input wave power [12,14]. The space-periodic $\delta$-functions represent small nonlinear dielectric regions that are periodically embedded in a linear dielectric medium. These nonlinear regions...
are assumed to be much smaller than the distance between adjacent ones. Quasiperiodicity in this model enters through the coefficients $\lambda_n$ that are assumed to follow the Fibonacci sequence. This sequence follows from the inflation rule: $S_{l+1} = S_l S_{l-1}$, where $S_0 = A$ and $S_1 = AB$. There are only two values of $\lambda_n$, $\lambda_A$ and $\lambda_B$ and the actual value of $\lambda_n$ at location $n$ is determined from the Fibonacci rule. We use the following procedure to obtain the sequence [15],

$$\mu_{n+1} = \lfloor \phi_n + 1/\gamma \rfloor,$$

$$\phi_{n+1} = (\phi_n + 1/\gamma) \mod(1),$$

where the square brackets denote the integer part, $\gamma = \frac{1}{2}(\sqrt{5} + 1)$ is the golden mean and we start with $\phi_0 = 0$ at $n = 1$. The value of $\lambda_n$ is $\lambda_A$ or $\lambda_B$ when $\mu_n$ is 1 or 0 respectively.

We investigate the scattering problem of a quasiparticle with momentum $k$. Plane waves are sent from the left towards the nonlinear chain and will be scattered into a reflected and transmitted part. On the left of the first $\delta$-function potential we have the incident and reflected waves

$$\psi(z) = R_0 \exp(ikz) + R \exp(-ikz),$$

and on the right end of the chain the transmitted wave

$$\psi(z) = T \exp(ikz).$$

Straightforward manipulations similar to the ones used in the standard linear Kronig–Penney problem lead to the following nonlinear difference equation for $\psi_n \equiv \psi(n)$ [15–17]

$$\psi_{n+1} + \psi_{n-1} = \frac{2 \cos(k)}{k} \sin(k) \left[ 2 \sin(k) + \frac{\lambda_n |\psi_n|^2 \sin(k)}{k} \right] \psi_n,$$

where $k$ is the wavenumber associated with the energy (or frequency) $E(k) = 2 \cos(k)$. Eq. (5) can be treated as a nonlinear map for various initial conditions corresponding to waves injected initially from the left and propagating towards the right side of the chain.

In Fig. 1 we show the intensity $|R_0|^2$ of a wave entering from the left as a function of the wavenumber $k$ for the linear Fibonacci Kronig–Penney model (Fig. 1a) and the nonlinear one (Fig. 1b). Dark regions represent gaps in the wave propagation whereas the passing regions are white. In the linear case, i.e. when the term $|\psi(z)|^2$ is absent from Eq. (1), we have the typical band structure resulting from quasiperiodicity [18]. We note that the effect of nonlinearity alters substantially this structure resulting in enhanced propagation for small initial intensities. The lattice is transparent for essentially all wavenumbers $k$ at low intensities. In particular, the dominant linear gap for $k$-values approximately less than $k \sim 2$ is reduced drastically. In the higher intensity region, on the other hand, the forbidden lines seem to coalesce together to form well defined nonlinear gaps that are interrupted periodically from propagating resonance-like zones. The latter occur for $k$-values that are multiples of $\pi$; for these wavenumbers the nonlinear term is effectively canceled leading to perfect propagation.

To analyze further the effects of nonlinearity in the
model we calculate the reduced transmission coefficient $t = |T|^2/|R_0|^2$, where $T$ is the transmitted amplitude at the end of the chain and $R_0$ is the injected amplitude at the beginning of the chain. In Fig. 2 we plot $t$ as a function of the chain length for the linear Fibonacci Kronig–Penney case for a typical gap state (Fig. 2a) and the corresponding nonlinear one (Fig. 2b). In the linear case the transmission drops exponentially with system length whereas in the nonlinear case we have almost perfect transmissivity for chain lengths in excess of one order of magnitude compared to the linear case. The enhancement in coherence resulting from the nonlinear term can be seen also in the correlation function plots of Fig. 3. We use polar coordinates $\psi_m = r_m \exp(i \theta_m)$ and define the phase correlation function $C(m)$ as

$$C(m) = (\theta_{m+1} \theta_m) = \frac{1}{N} \sum_{n=1}^{N} \theta_n \theta_{n+m},$$

where $m$ is a lattice site. In Fig. 3a we see the quick drop in the phase correlations in the linear Fibonacci
chain while in the nonlinear case (Fig. 3b) the correlation function is nondecaying and oscillates persistently.

The main result of this Letter is related to the enhancement at low wave intensities of the propagating properties of a quasiperiodic Kronig-Penney type model when nonlinearity is added to the model. Nonlinearity in the form contained in Eq. (1) assists the waves in defying the quasi-random properties of the medium. This effect seems to be also present in genuine nonlinear and disorder segments but with some differences [19]. The nonlinear lattice model we presented here has applications in the propagation of electrons in superlattices and electromagnetic waves in dielectric materials. In the latter case a quasiperiodic linear term should also be included in Eq. (1).
representing the linear dielectric constant of the material. The presence of this term does not qualitatively change the results of this Letter [12].

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References